ROMANIAN MATHEMATICAL MAGAZINE

In $\Delta ABC$ the following relationship holds:

$$h_a h_b h_c \geq 27r^3$$

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$$h_a h_b h_c = \frac{2F}{a} \cdot \frac{2F}{b} \cdot \frac{2F}{c} = \frac{8F^3}{abc} = \frac{8F^3}{4RF} = \frac{2F^2}{R} = \frac{2r^2s^2}{R}$$

We must prove that:

$$\frac{2r^2s^2}{R} \geq 27r^3 \Leftrightarrow \frac{2s^2}{R} \geq 27r \Leftrightarrow 27r \Leftrightarrow 2s^2 \geq 27Rr$$

$$2s^2 \overset{GERRETSEN}{\geq} 2(16Rr - 5r^2) \geq 27Rr$$

$$32Rr - 10r^2 \geq 27Rr$$

$$5Rr \geq 10r^2$$

$$R \geq 2r \quad \text{(Euler)}$$

Equality holds for $a = b = c$.