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In $\Delta ABC$ the following relationship holds:

$$\frac{r_a}{b}+\frac{r_b}{c}+\frac{r_c}{a}\ge 3\sqrt{3}\cdot\frac{r}{R}$$

*Proposed by Nguyen Hung Cuong – Vietnam*

*Solution by Daniel Sitaru – Romania*

$$\frac{r_a}{b}+\frac{r_b}{c}+\frac{r_c}{a}=\sum_{cyc}\frac{r_a}{b}=\sum_{cyc}\frac{F}{b(s-a)}=$$

$$=F\sum_{cyc}\frac{1}{b(s-a)}\overset{AM-GM}{\ge}\frac{3F}{\sqrt[3]{abc(s-a)(s-b)(s-c)}}=$$

$$=\frac{3F}{\sqrt[3]{4Rrs(s-a)(s-b)(s-c)}}=\frac{3F}{\sqrt[3]{4RrF^2}}=$$

$$=\frac{3F}{\sqrt[3]{4Rrr^2s^2}}=\frac{3F}{r\cdot\sqrt[3]{4Rs^2}}=\frac{3rs}{r\cdot\sqrt[3]{4Rs^2}}\overset{MITRINOVIC}{\ge}\frac{3s}{\sqrt[3]{4R\left(\frac{3\sqrt{3}R}{2}\right)^2}}\overset{MITRINOVIC}{\ge}$$

$$\ge\frac{3\cdot 3\sqrt{3}r}{\sqrt[3]{4R\cdot\frac{27R^2}{4}}}=\frac{9\sqrt{3}r}{3R}=3\sqrt{3}\cdot\frac{r}{R}$$

Equality holds for $a=b=c$.