ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{r_a^2}{h_b}+\frac{r_b^2}{h_c}+\frac{r_c^2}{h_a}\ge 9r$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{r_a^2}{h_b}+\frac{r_b^2}{h_c}+\frac{r_c^2}{h_a}\overbrace{\ge}^{Bergstrom}\frac{(r_a+r_b+r_c)^2}{h_a+h_b+h_c}=\frac{(r_a+r_b+r_c)^2}{\frac{ac}{2R}+\frac{ab}{2R}+\frac{bc}{2R}}=\frac{2R(r_a+r_b+r_c)^2}{ab+bc+ac}\ge$$

$$\ge\frac{3.2R(4R+r)^2}{(a+b+c)^2}=\frac{6R(4R+r)^2}{4s^2}\overbrace{\ge}^{Mitrinovici}\frac{6.\frac{2s}{3\sqrt{3}}(4R+r)^2}{4s^2}=$$

$$=\frac{(4R+r)(4R+r)}{s\sqrt{3}}\overbrace{\ge}^{Doucet}4R+r\overbrace{\ge}^{Euler}9r$$

$$Equality\ \ holds\ \ for\ \ a=b=c.$$