ROMANIAN MATHEMATICAL MAGAZINE

Prove that in ΔABC the following relationship holds:

$$\frac{a}{r_b}+\frac{b}{r_c}+\frac{c}{r_a}\geq 2\sqrt{3}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{a}{r_b}+\frac{b}{r_c}+\frac{c}{r_a}\overset{AM-GM}{\geq}3\left(\frac{abc}{r_ar_br_c}\right)^{\frac{1}{3}}=$$

$$=3\left(\frac{4RF}{Fp}\right)^{\frac{1}{3}}=3\left(\frac{4R}{p}\right)^{\frac{1}{3}}\overset{Mitrinovic}{\geq}3\left(\frac{4}{p}\cdot\frac{2p}{3\sqrt{3}}\right)^{\frac{1}{3}}=\frac{6}{\sqrt{3}}=2\sqrt{3}$$

$$Equality\ \ holds\ \ for\ \ a=b=c.$$