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In $\triangle ABC$ the following relationship holds:

$$\frac{ab}{r_c^2} + \frac{bc}{r_a^2} + \frac{ac}{r_b^2} \geq 4$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \frac{ab}{r_c^2} + \frac{bc}{r_a^2} + \frac{ac}{r_b^2} &\stackrel{AM-GM}{\geq} 3 \left(\left(\frac{abc}{r_a r_b r_c} \right)^2 \right)^{\frac{1}{3}} = \\ &= 3 \left(\left(\frac{4RF}{Fp} \right)^2 \right)^{\frac{1}{3}} = 3 \left(\frac{16R^2}{p^2} \right)^{\frac{1}{3}} \stackrel{Mitrinovic}{\geq} 3 \left(\frac{16}{p^2} \cdot \frac{4p^2}{27} \right)^{\frac{1}{3}} = 4 \end{aligned}$$

Equality holds for $a = b = c$.