ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC the following relationship holds :

$$\sum_{cyc} \frac{\sqrt{\cot A}}{a} \le \frac{1}{2r^2} \cdot \sqrt{\frac{2R^2 + r^2}{3\sqrt{3}}}$$

Proposed by Marin Chirciu-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc} \frac{\sqrt{\cot A}}{a} \overset{\text{CBS}}{\le} \sqrt{\sum_{cyc} \cot A} \cdot \sqrt{\frac{1}{16R^2r^2s^2} \cdot \sum_{cyc} a^2b^2} \overset{\text{Goldstone}}{\le}$$

$$\sqrt{\frac{s^2 - 4Rr - r^2}{2rs}} \cdot \sqrt{\frac{4R^2s^2}{16R^2r^2s^2}} \overset{\substack{\text{Mitrinovic} \\ \text{and} \\ \text{Gerretsen}}}{\le} \sqrt{\frac{4R^2 + 2r^2}{2r^2 . 3\sqrt{3}}} \cdot \sqrt{\frac{1}{4r^2}} = \frac{1}{2r^2} \cdot \sqrt{\frac{2R^2 + r^2}{3\sqrt{3}}}$$

and so, $\displaystyle\sum_{cyc} \frac{\sqrt{\cot A}}{a} \le \frac{1}{2r^2} \cdot \sqrt{\frac{2R^2 + r^2}{3\sqrt{3}}}$ $\forall \Delta$ ABC, " = " iff Δ ABC is equilateral (QED)