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In $\triangle ABC$ the following relationship holds:

$$\sum \frac{h_a}{bc} \leq \frac{3}{4r}$$

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Let Δ denote the area of $\triangle ABC$.

$$\begin{aligned} \text{Here, } \sum \frac{h_a}{bc} &= \sum \frac{\frac{2\Delta}{a}}{bc} = \sum \frac{2\Delta}{abc} = \sum \frac{2\Delta}{4\Delta \cdot R} = \sum \frac{1}{2R} = \\ &= \frac{3}{2R} \stackrel{\text{Euler}}{\leq} \frac{3}{2 \cdot 2r} = \frac{3}{4r} \end{aligned}$$

Equality holds iff the triangle is equilateral. (QED)