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In any ΔABC the following relationship holds :

$$\sum_{\text{cyc}} \frac{\tan \frac{A}{2}}{\tan^2 \frac{B}{2}} \geq 3\sqrt{3}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} \sum_{\text{cyc}} \frac{\tan \frac{A}{2}}{\tan^2 \frac{B}{2}} &= s \sum_{\text{cyc}} \frac{r_a}{r_b^2} = s \cdot \sum_{\text{cyc}} \frac{\left(\frac{1}{r_b}\right)^2}{\frac{1}{r_a}} \stackrel{\text{Bergstrom}}{\geq} s \cdot \frac{\left(\sum_{\text{cyc}} \frac{1}{r_a}\right)^2}{\sum_{\text{cyc}} \frac{1}{r_a}} = \frac{s \cdot \frac{1}{r^2}}{\frac{1}{r}} = \frac{s}{r} \\ &\stackrel{\text{Mitrinovic}}{\geq} 3\sqrt{3} \text{ and so, } \sum_{\text{cyc}} \frac{\tan \frac{A}{2}}{\tan^2 \frac{B}{2}} \geq 3\sqrt{3} \quad \forall \Delta ABC, \end{aligned}$$

" = " iff ΔABC is equilateral (QED)