ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$3 \leq \sqrt{\sum_{cyc} m_a \cdot \sum_{cyc} \frac{1}{m_a}} \leq \frac{3R}{2r}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc} m_a \cdot \sum_{cyc} \frac{1}{m_a} \geq 9 \Rightarrow \sqrt{\sum_{cyc} m_a \cdot \sum_{cyc} \frac{1}{m_a}} \geq 3$$

$$\sum_{cyc} m_a \leq \frac{9R}{2} \quad (Gotman)$$

$$m_a \geq h_a \Rightarrow \frac{1}{m_a} \leq \frac{1}{h_a} \Rightarrow \sum_{cyc} \frac{1}{m_a} \leq \sum_{cyc} \frac{1}{h_a} = \frac{1}{r}$$

$$\sqrt{\sum_{cyc} m_a \cdot \sum_{cyc} \frac{1}{m_a}} \leq \sqrt{\frac{9R}{2} \cdot \frac{1}{r}} \leq \frac{3R}{2r} \Leftrightarrow \frac{9R}{2r} \leq \frac{9R^2}{4r^2} \Leftrightarrow 2r \leq R \ (Euler)$$

$$3 \leq \sqrt{\sum_{cyc} m_a \cdot \sum_{cyc} \frac{1}{m_a}} \leq \frac{3R}{2r}$$

$$Equality \ holds \ for \ a = b = c.$$