ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{12}{a+b+c} \le \sum \frac{a}{r_a^2} \le \frac{4}{a+b+c}\left(\frac{2R}{r}-1\right)$$

Proposed by Marin Chirciu-Romania

Solution by Tapas Das-India

$$\sum \frac{a}{r_a^2} = \frac{1}{F^2}\sum a(s-a)^2 = \frac{1}{F^2}\sum a(s^2-2sa+a^2) =$$

$$= \frac{1}{r^2s^2}\left(2s^3-4s(s^2-r^2-4Rr)+2s(s^2-3r^2-6Rr)\right) =$$

$$= \frac{2s(2Rr-r^2)}{r^2s^2} = \frac{4}{2s}\left(\frac{2Rr-r^2}{r^2}\right) = \frac{4}{a+b+c}\left(\frac{2R}{r}-1\right)$$

$$\sum \frac{a}{r_a^2} = \frac{4}{a+b+c}\left(\frac{2R}{r}-1\right) \overset{EULER}{\ge} \frac{4}{a+b+c}(2\times 2-1) = \frac{12}{a+b+c}$$

$$\textit{Equality holds for an equilateral triangle.}$$