

# ROMANIAN MATHEMATICAL MAGAZINE

In any  $\Delta ABC$  the following relationship holds :

$$\left( \sum_{\text{cyc}} \frac{m_a}{w_b + w_c} \right) \left( \sum_{\text{cyc}} \frac{w_b + w_c}{\sqrt{r_b r_c}} \right) \geq 9$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} & \left( \sum_{\text{cyc}} \frac{m_a}{w_b + w_c} \right) \left( \sum_{\text{cyc}} \frac{w_b + w_c}{\sqrt{r_b r_c}} \right) \stackrel{\text{Reverse CBS}}{\geq} \left( \sum_{\text{cyc}} \frac{\sqrt{m_a}}{\sqrt[4]{r_b r_c}} \right)^2 \stackrel{\text{AM-GM}}{\geq} \\ & 9 \cdot \left( \frac{\sqrt{m_a m_b m_c}}{\sqrt[4]{s(s-a) \cdot s(s-b) \cdot s(s-c)}} \right)^{\frac{2}{3}} \stackrel{\text{Lascu} + \text{AM-GM}}{\geq} 9 \cdot \left( \frac{\sqrt[4]{s(s-a) \cdot s(s-b) \cdot s(s-c)}}{\sqrt[4]{s(s-a) \cdot s(s-b) \cdot s(s-c)}} \right)^{\frac{2}{3}} \\ & = 9 \text{ and so, } \left( \sum_{\text{cyc}} \frac{m_a}{w_b + w_c} \right) \left( \sum_{\text{cyc}} \frac{w_b + w_c}{\sqrt{r_b r_c}} \right) \geq 9 \forall \Delta ABC, \\ & \text{" = " iff } \Delta ABC \text{ is equilateral (QED)} \end{aligned}$$