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In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{m_a}{b+c}\cdot\sum_{cyc}\frac{b+c}{\sqrt{r_b r_c}}\ge 9$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{m_a}{b+c}\cdot\sum_{cyc}\frac{b+c}{\sqrt{r_b r_c}}\overset{CSB}{\ge}\left(\sum_{cyc}\sqrt{\frac{m_a}{b+c}}\cdot\sqrt{\frac{b+c}{\sqrt{r_b r_c}}}\right)^2=$$

$$=\left(\sum_{cyc}\sqrt{\frac{m_a}{\sqrt{r_b r_c}}}\right)^2\overset{AM-GM}{\ge}\left(3\left(\left(\frac{m_a m_b m_c}{r_a r_b r_c}\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^2\overset{m_a\ge\sqrt{s(s-a)}}{\ge}$$

$$\ge 9\left(\frac{s\sqrt{s(s-a)(s-b)(s-c)}}{sF}\right)^{\frac{1}{3}}=9\left(\frac{sF}{sF}\right)^{\frac{1}{3}}=9$$

$$Equality\ \ holds\ \ for\ \ a=b=c.$$