

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\sqrt{\frac{w_a}{r_a}} + \sqrt{\frac{r_a}{w_a}} \leq \sqrt{\frac{2R}{r}}$$

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$$n_a^2 = s^2 - 2r_a h_a \text{ (Bogdan Fustei) }, r_a = s \tan \frac{A}{2}$$

$$\begin{aligned} \sqrt{\frac{w_a}{r_a}} + \sqrt{\frac{r_a}{w_a}} &= \frac{w_a + r_a}{\sqrt{w_a r_a}} = \sqrt{\frac{(w_a + r_a)^2}{w_a r_a}} = \sqrt{\frac{w_a^2 + r_a^2}{w_a r_a} + 2} \stackrel{\substack{w_a \leq n_a \\ w_a \geq ha}}{\leq} \sqrt{\frac{n_a^2 + r_a^2}{h_a r_a} + 2} = \\ &= \sqrt{\frac{s^2 - 2r_a h_a + r_a^2}{r_a h_a} + 2} = \sqrt{\frac{s^2 + s^2 \tan^2 \left(\frac{A}{2}\right)}{\frac{2F^2}{a(s-a)}}} = \sqrt{\frac{s^2 \sec^2 \left(\frac{A}{2}\right)}{\frac{2F^2}{a(s-a)}}} = \sqrt{\frac{a(s-a)}{2r^2 \cos^2 \left(\frac{A}{2}\right)}} = \\ &= \sqrt{\frac{abc}{2sr^2}} = \sqrt{\frac{4Rrs}{2sr^2}} = \sqrt{\frac{2R}{r}} \end{aligned}$$

Equality holds for an equilateral triangle.