ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sqrt{\frac{m_a}{r_a}}+\sqrt{\frac{r_a}{m_a}}\le\sqrt{\frac{2R}{r}}$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

$$n_a^2=s^2-2r_ah_a(Bogdan\ Fustei)\ ,r_a=s\tan\frac{A}{2}$$

$$\sqrt{\frac{m_a}{r_a}}+\sqrt{\frac{r_a}{m_a}}=\frac{m_a+r_a}{\sqrt{m_ar_a}}=\sqrt{\frac{(m_a+r_a)^2}{m_ar_a}}=\sqrt{\frac{m_a^2+r_a^2}{m_ar_a}+2}\overset{\substack{m_a\le n_a\\ m_a\ge ha}}{\le}\sqrt{\frac{n_a^2+r_a^2}{h_ar_a}+2}=$$

$$=\sqrt{\frac{s^2-2r_ah_a+r_a^2}{r_ah_a}+2}=\sqrt{\frac{s^2+s^2tan^2\left(\frac{A}{2}\right)}{\frac{2F^2}{a(s-a)}}}=\sqrt{\frac{s^2sec^2\left(\frac{A}{2}\right)}{\frac{2F^2}{a(s-a)}}}=\sqrt{\frac{a(s-a)}{2r^2cos^2\left(\frac{A}{2}\right)}}=$$

$$=\sqrt{\frac{abc}{2sr^2}}=\sqrt{\frac{4Rrs}{2sr^2}}=\sqrt{\frac{2R}{r}}$$

Equality holds for an equilateral triangle.