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In any Δ ABC the following relationship holds :

$$\frac{r_b+r_c}{w_a}\le\frac{\sin\frac{B}{2}}{\sin\frac{C}{2}}+\frac{\sin\frac{C}{2}}{\sin\frac{B}{2}}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\frac{r_b+r_c}{w_a}\overset{?}{\le}\frac{\sin\frac{B}{2}}{\sin\frac{C}{2}}+\frac{\sin\frac{C}{2}}{\sin\frac{B}{2}}\Leftrightarrow\frac{4R.s(s-a)}{bc.\frac{2bc}{b+c}.\cos^2\frac{A}{2}}\overset{?}{\le}\frac{\frac{(s-c)(s-a)}{ca}+\frac{(s-a)(s-b)}{ab}}{\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}}$$

$$\Leftrightarrow\frac{s(s-a)}{bc.\frac{2bc}{b+c}.\frac{s(s-a)}{bc}}\overset{?}{\le}\frac{b(s-c)(s-a)+c(s-a)(s-b)}{abc.(s-a)}$$

$$\Leftrightarrow 2\big(zx(z+x)+xy(x+y)\big)\overset{?}{\ge}x(y+z)(2x+y+z)$$

$$(x=s-a,y=s-b,z=s-c\Rightarrow a=y+z,b=z+x,c=x+y)$$

$$\Leftrightarrow x(y-z)^2\overset{?}{\ge}0\rightarrow\text{true}\because x>0\therefore\frac{r_b+r_c}{w_a}\le\frac{\sin\frac{B}{2}}{\sin\frac{C}{2}}+\frac{\sin\frac{C}{2}}{\sin\frac{B}{2}}\ \forall\Delta\,ABC,$$

$$''='' \text{ iff } b=c$$