ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{w_a}{h_a}+\frac{h_a}{w_a}\le\sqrt{\frac{R+6r}{2r}}$$

Proposed by Dang Ngoc Minh-Vietnam

Solution by Tapas Das-India

$$\frac{w_a}{h_a}=\frac{2\sqrt{bc.s(s-a)}}{b+c}.\frac{2R}{bc}=\frac{4R\sqrt{s(s-a)}}{(2(s-a)+a)\sqrt{bc}}\overset{AM-GM}{\le}\frac{4R\sqrt{s(s-a)}}{2\sqrt{2(s-a)abc}}=$$

$$=2R\sqrt{\frac{s}{8Rsr}}=\sqrt{\frac{R}{2r}}(1)$$

$$\left(\frac{w_a}{h_a}+\frac{h_a}{w_a}\right)^2=\left(\frac{w_a}{h_a}\right)^2+\left(\frac{h_a}{w_a}\right)^2+2\overset{\substack{(1)\\ w_a\ge h_a}}{\le}\left(\sqrt{\frac{R}{2r}}\right)^2+\left(\frac{w_a}{w_a}\right)^2+2=\frac{R+6r}{2r}$$

$$\frac{w_a}{h_a}+\frac{h_a}{w_a}\le\sqrt{\frac{R+6r}{2r}}$$

Equality holds for an equilateral triangle.