ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC with $a = \max\{a, b, c\}$, the following relationship holds :

$$\frac{a}{b+c} \geq 1 - \frac{r}{R}$$

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$$\frac{a}{b+c} \overset{?}{\geq} 1 - \frac{r}{R} \Leftrightarrow \frac{4(s-a)(s-b)(s-c)}{abc} \overset{?}{\geq} \frac{2(s-a)}{b+c}$$

$$\Leftrightarrow 2yz(2x+y+z) \overset{?}{\geq} (y+z)(z+x)(x+y)$$

$$\left(\begin{matrix} x = s-a, y = s-b, z = s-c \Rightarrow a = y+z, b = z+x, c = x+y \text{ and} \\ s = x+y+z \end{matrix}\right)$$

$$\Leftrightarrow y^2(z-x) + z^2(y-x) + xy(z-x) + zx(y-x) \overset{?}{\geq} 0 \to \text{true}$$

$$\because a \geq b, c \Rightarrow y, z \geq x \therefore \frac{a}{b+c} \geq 1 - \frac{r}{R} \ \forall \Delta \text{ ABC with } a = \max\{a, b, c\},$$

$$'' ='' \text{ iff } \Delta \text{ ABC is equilateral (QED)}$$