ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC the following relationship holds :

$$1 \geq \frac{2m_a - g_a - \sqrt{4r^2 + (b-c)^2}}{n_a} + \frac{2m_b - g_b - \sqrt{4r^2 + (c-a)^2}}{n_b} + \frac{2m_c - g_c - \sqrt{4r^2 + (a-b)^2}}{n_c}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\sum_{cyc} \frac{2m_a - g_a - \sqrt{4r^2 + (b-c)^2}}{n_a} \overset{\text{Bogdan Fustei}}{\leq}$$

$$\sum_{cyc} \frac{n_a + g_a - g_a - \sqrt{4r^2 + (b-c)^2}}{n_a} \overset{\text{Bogdan Fustei}}{=} \sum_{cyc} \frac{n_a - \frac{an_a}{s}}{n_a} = \sum_{cyc} \left(1 - \frac{a}{s}\right)$$

$$= 3 - \frac{2s}{s} = 1 \text{ and so, } 1 \geq \frac{2m_a - g_a - \sqrt{4r^2 + (b-c)^2}}{n_a} +$$

$$\frac{2m_b - g_b - \sqrt{4r^2 + (c-a)^2}}{n_b} + \frac{2m_c - g_c - \sqrt{4r^2 + (a-b)^2}}{n_c} \ \forall \Delta \text{ ABC},$$

$$'' ='' \text{ iff } \Delta \text{ ABC is equilateral (QED)}$$