

ROMANIAN MATHEMATICAL MAGAZINE

In any ΔABC the following relationship holds :

$$R \prod_{\text{cyc}} (s + n_a)(w_a + w_b + m_c - n_a \cdot \sqrt{3}) \leq 48\sqrt{3}r(r_a r_b r_c)^2$$

Proposed by Bogdan Fuștei-Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned}
 & R \prod_{\text{cyc}} (s + n_a)(w_a + w_b + m_c - n_a \cdot \sqrt{3}) \stackrel{\text{Lessel-Pelling}}{\leq} \\
 & R \prod_{\text{cyc}} (s + n_a)(s \cdot \sqrt{3} - n_a \cdot \sqrt{3}) = R \cdot 3\sqrt{3} \cdot \prod_{\text{cyc}} (s^2 - n_a^2) \stackrel{\text{Bogdan Fustei}}{=} \\
 & 8R \cdot 3\sqrt{3} \cdot \prod_{\text{cyc}} (h_a r_a) = 24\sqrt{3} \cdot \frac{2r^2 s^2}{R} \cdot (r_a r_b r_c) = 48\sqrt{3}r \cdot (r_a r_b r_c)^2 (\because rs^2 = r_a r_b r_c) \\
 & \text{and so, } R \prod_{\text{cyc}} (s + n_a)(w_a + w_b + m_c - n_a \cdot \sqrt{3}) \leq 48\sqrt{3}r(r_a r_b r_c)^2 \forall \Delta ABC, \\
 & \quad \quad \quad " = " \text{ iff } \Delta ABC \text{ is equilateral (QED)}
 \end{aligned}$$