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In any ΔABC the following relationship holds :

$$2\sqrt{3} \cdot \sum_{\text{cyc}} \frac{h_a}{w_a + w_b + m_c - n_a \cdot \sqrt{3}} \geq \frac{s}{r} + \sum_{\text{cyc}} \frac{n_a}{r_a}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\begin{aligned} & 2\sqrt{3} \sum_{\text{cyc}} \frac{h_a}{w_a + w_b + m_c - n_a \cdot \sqrt{3}} \stackrel{\text{Lessel-Pelling}}{\geq} 2\sqrt{3} \cdot \sum_{\text{cyc}} \frac{h_a}{s \cdot \sqrt{3} - n_a \cdot \sqrt{3}} \\ &= \frac{2\sqrt{3}}{\sqrt{3}} \cdot \sum_{\text{cyc}} \frac{h_a(n_a + s)}{s^2 - n_a^2} \stackrel{\text{Bogdan Fustei}}{=} 2 \sum_{\text{cyc}} \frac{h_a(s + n_a)}{2h_a r_a} = s \sum_{\text{cyc}} \frac{1}{r_a} + \sum_{\text{cyc}} \frac{n_a}{r_a} \\ &= \frac{s}{r} + \sum_{\text{cyc}} \frac{n_a}{r_a} \therefore 2\sqrt{3} \cdot \sum_{\text{cyc}} \frac{h_a}{w_a + w_b + m_c - n_a \cdot \sqrt{3}} \geq \frac{s}{r} + \sum_{\text{cyc}} \frac{n_a}{r_a} \forall \Delta ABC, \end{aligned}$$

" = " iff ΔABC is equilateral (QED)