ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\cot A + \cot B + \cot C = \frac{\sin^2 A + \sin^2 B + \sin^2 C}{2 \sin A \sin B \sin C}$$

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Solution by Daniel Sitaru – Romania

$$\cot A + \cot B + \cot C = \sum_{cyc} \cot A = \sum_{cyc} \frac{\cos A}{\sin A} =$$

$$= \sum_{cyc} \frac{\frac{b^2 + c^2 - a^2}{2bc}}{\sin A} = \sum_{cyc} \frac{b^2 + c^2 - a^2}{2bc \sin A} = \sum_{cyc} \frac{b^2 + c^2 - a^2}{4F} =$$

$$= \frac{1}{4F}\left(\sum_{cyc} b^2 + \sum_{cyc} c^2 - \sum_{cyc} a^2\right) = \frac{1}{4F}\left(\sum_{cyc} a^2 + \sum_{cyc} a^2 - \sum_{cyc} a^2\right) = \frac{1}{4F}\sum_{cyc} a^2 =$$

$$= \frac{1}{4F}(a^2 + b^2 + c^2) = \frac{1}{4 \cdot 2R^2 \sin A \sin B \sin C}\sum_{cyc} 4R^2 \sin^2 A =$$

$$= \frac{4R^2}{8R^2} \cdot \frac{\sin^2 A + \sin^2 B + \sin^2 C}{\sin A \sin B \sin C} = \frac{\sin^2 A + \sin^2 B + \sin^2 C}{2 \sin A \sin B \sin C}$$