

# ROMANIAN MATHEMATICAL MAGAZINE

If in  $\triangle ABC$  we have:  $\tan \frac{A}{2} \tan \frac{B}{2} = \frac{1}{3}$  then:

$$2 \sin C = \sin A + \sin B$$

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*Solution by Daniel Sitaru – Romania*

$$\tan \frac{A}{2} \tan \frac{B}{2} = \frac{1}{3} \Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \cdot \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} = \frac{1}{3}$$

$$\Rightarrow \frac{s-c}{s} = \frac{1}{3} \Rightarrow 3s - 3c = s \Rightarrow 2s - 3c = 0$$

$$\Rightarrow a + b + c - 3c = 0 \Rightarrow 2c = a + b$$

$$2 \cdot 2R \sin C = 2R \sin A + 2R \sin B$$

$$2 \sin C = \sin A + \sin B$$