ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sin 2026A + \sin 2026B + \sin 2026C = 4\sin 1013A \sin 1013B \sin 1013C$$

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Denote:

$$1013A = x; 1013B = y; 1013C = z$$

$$x + y + z = 1013(A + B + C) = 1013\pi \quad (1)$$

$$\cos\left(\frac{1013\pi}{2} - y\right) = \cos\frac{1013\pi}{2}\cos y + \sin\frac{1013\pi}{2}\sin y =$$

$$= \cos\left(506\pi + \frac{\pi}{2}\right)\cos y + \sin\left(506\pi + \frac{\pi}{2}\right)\sin y = \cos\frac{\pi}{2}\cos y + \sin\frac{\pi}{2}\sin y = \sin y$$

$$\cos\left(\frac{1013\pi}{2} - y\right) = \sin y \quad (2)$$

Analogous:

$$\cos\left(x - \frac{1013\pi}{2}\right) = \cos\left(\frac{1013\pi}{2} - x\right) = \sin x \quad (3)$$

$$\sin 2026A + \sin 2026B + \sin 2026C = \sin 2x + \sin 2y + \sin 2z =$$

$$= 2\sin\frac{2x + 2y}{2}\cos\frac{2x - 2y}{2} + 2\sin z\cos z =$$

$$= 2\sin(x + y)\cos(x - y) + 2\sin z\cos z \overset{(1)}{=}$$

$$= 2\sin(1013\pi - z)\cos(x - y) + 2\sin z\cos z =$$

$$= 2(\sin 1013\pi\cos z - \sin z\cos 1013\pi)\cos(x - y) + 2\sin z\cos z =$$

$$= 2(0 \cdot \cos z - \sin z \cdot (-1)^{1013})\cos(x - y) + 2\sin z\cos z =$$

$$= 2\sin z\cos(x - y) + 2\sin z\cos z = 2\sin z\left(\cos(x - y) + \cos z\right) =$$

$$= 2\sin z \cdot 2\cos\frac{x - y + z}{2} \cdot \cos\frac{x - y - z}{2} =$$

$$= 4\sin z\cos\frac{x + z - y}{2}\cos\frac{y + z - x}{2} \overset{(1)}{=}$$

$$= 4\sin z\cos\left(\frac{1013\pi - 2y}{2}\right)\cos\left(\frac{1013\pi - 2x}{2}\right) \overset{(2),(3)}{=} 4\sin z\sin y\sin x =$$

$$= 4\sin 1013A\sin 1013B\sin 1013C$$