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In ΔABC the following relationship holds:

$$\frac{\sin\frac{A}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}}+\frac{\sin\frac{B}{2}}{\cos\frac{C}{2}\cos\frac{A}{2}}+\frac{\sin\frac{C}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}}=2$$

Proposed by Nguyen Hung Cuong – Vietnam

Solution by Daniel Sitaru – Romania

$$\frac{\sin\frac{A}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}}+\frac{\sin\frac{B}{2}}{\cos\frac{C}{2}\cos\frac{A}{2}}+\frac{\sin\frac{C}{2}}{\cos\frac{A}{2}\cos\frac{B}{2}}=$$

$$=\sum_{cy}\frac{\sin\frac{A}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}}=\sum_{cyc}\sqrt{\frac{(s-b)(s-c)}{bc}}\cdot\sqrt{\frac{ac\cdot ab}{s(s-b)\cdot s(s-c)}}=$$

$$=\sum_{cyc}\sqrt{\frac{a^2}{s^2}}=\frac{1}{s}\sum_{cyc}a=\frac{a+b+c}{s}=\frac{2s}{s}=2$$