

ROMANIAN MATHEMATICAL MAGAZINE

If in $\triangle ABC$ we have:

$$\tan \frac{A}{2} \tan \frac{B}{2} = \frac{1}{5}$$

then:

$$3\sin C = 2\sin A + 2\sin B$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\tan \frac{A}{2} \tan \frac{B}{2} = \frac{1}{5} \Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \cdot \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} = \frac{1}{5} \Rightarrow$$

$$\Rightarrow \frac{s-c}{s} = \frac{1}{5} \Rightarrow 5s - 5c = s \Rightarrow 4s = 5c \Rightarrow$$

$$\Rightarrow 2a + 2b + 2c = 5c \Rightarrow 3c = 2a + 2b \Rightarrow$$

$$\Rightarrow 3 \cdot 2R\sin C = 2 \cdot 2R\sin A + 2 \cdot 2R\sin B \Rightarrow 3\sin C = 2\sin A + 2\sin B$$