ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$w_a\cos\left(\frac{A}{2}\right)+w_b\cos\left(\frac{B}{2}\right)+w_c\cos\left(\frac{C}{2}\right)=\frac{sr}{R}\left(\frac{r_a+r_b}{h_a+h_b}+\frac{r_c+r_b}{h_c+h_b}+\frac{r_a+r_c}{h_a+h_c}\right)$$

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$$r_c+r_b=s\left(\tan\left(\frac{B}{2}\right)+\tan\left(\frac{C}{2}\right)\right)=s.\frac{\sin\left(\frac{B+C}{2}\right)}{\cos\left(\frac{B}{2}\right).\cos\left(\frac{C}{2}\right)}=$$

$$s.\frac{\cos\left(\frac{A}{2}\right)}{\cos\left(\frac{B}{2}\right).\cos\left(\frac{C}{2}\right)}=s.\frac{cos^2\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right).\cos\left(\frac{B}{2}\right).\cos\left(\frac{C}{2}\right)}=s.\frac{cos^2\left(\frac{A}{2}\right)}{\frac{s}{4R}}=4R.cos^2\left(\frac{A}{2}\right)\quad(1)$$

$$h_c+h_b=\frac{2F(b+c)}{bc}\quad(2)$$

We have from (1) and (2):

$$\frac{r_c+r_b}{h_c+h_b}=\frac{4R.cos^2\left(\frac{A}{2}\right)}{\frac{2F(b+c)}{bc}}=\frac{4Rbc.cos^2\left(\frac{A}{2}\right)}{2sr(b+c)}=$$

$$\frac{2bc.\cos\left(\frac{A}{2}\right)}{b+c}.\cos\left(\frac{A}{2}\right)=w_a\cos\left(\frac{A}{2}\right).\frac{R}{sr}$$

$$w_a\cos\left(\frac{A}{2}\right)=\frac{sr}{R}.\frac{r_c+r_b}{h_c+h_b}$$

Let's summarize what we got from others by analogy:

$$w_a\cos\left(\frac{A}{2}\right)+w_b\cos\left(\frac{B}{2}\right)+w_c\cos\left(\frac{C}{2}\right)=\frac{sr}{R}\left(\frac{r_a+r_b}{h_a+h_b}+\frac{r_c+r_b}{h_c+h_b}+\frac{r_a+r_c}{h_a+h_c}\right)$$