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In $\triangle ABC$ the following relationship holds:

$$h_a + h_b + h_c = 9r + r \sum_{cyc} \left(\sqrt{\frac{b}{c}} - \sqrt{\frac{c}{b}} \right)^2$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} h_a + h_b + h_c &= 9r + r \sum_{cyc} \left(\sqrt{\frac{b}{c}} - \sqrt{\frac{c}{b}} \right)^2 \\ \frac{2F}{a} + \frac{2F}{b} + \frac{2F}{c} &= \frac{9F}{s} + \frac{F}{s} \sum_{cyc} \left(\frac{b}{c} + \frac{c}{b} - 2 \right) \\ 2 \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) &= \frac{3}{s} + \frac{1}{s} \sum_{cyc} \left(\frac{b}{c} + \frac{c}{b} \right) \\ 2s \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) &= 3 + \sum_{cyc} \left(\frac{b}{c} + \frac{c}{b} \right) \\ (a + b + c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) &= 3 + \sum_{cyc} \left(\frac{b}{c} + \frac{c}{b} \right) \\ 3 + \sum_{cyc} \left(\frac{b}{c} + \frac{c}{b} \right) &= 3 + \sum_{cyc} \left(\frac{b}{c} + \frac{c}{b} \right) \end{aligned}$$