ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z>0$ $xy+yz+zx=3$ then:

$$\sum_{cyc}\frac{x}{2x^4+y^2+z^2}\le\frac{3}{4xyz}$$

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$$2x^4+y^2+z^2=x^4+x^4+y^2+z^2\overset{AM-GM}{\underbrace{\ge}}$$

$$\ge 4(x^4.x^4.y^2.z^2)^{\frac{1}{4}}=4x^2\sqrt{yz}$$

$$Analogously\ :\ \begin{cases}2y^4+x^2+z^2\ge 4y^2\sqrt{xz}\\ 2z^4+y^2+x^2\ge 4z^2\sqrt{yx}\end{cases}\ (*)$$

$$\sum_{cyc}\frac{x}{2x^4+y^2+z^2}\overset{(*)}{\underbrace{\le}}\sum_{cyc}\frac{x}{4x^2\sqrt{yz}}=\sum_{cyc}\frac{1}{4x\sqrt{yz}}=$$

$$=\frac{\sqrt{xy}+\sqrt{yz}+\sqrt{zx}}{4xyz}\le\frac{\sqrt{3(xy+yz+zx)}}{4xyz}=\frac{\sqrt{3.3}}{4xyz}=\frac{3}{4xyz}$$

$$Equality\ \ holds\ \ for\ \ x=y=z=1.$$