ROMANIAN MATHEMATICAL MAGAZINE

If $x_i > 0 \quad i \in \{1,2,3,\dots,n\}, \qquad x_1 + x_2 + \dots + x_n = 1$ then prove :

$$\sum_{k=1}^{n} \frac{1}{\sqrt{1+x_k}} \geq n\sqrt{\frac{n}{n+1}}$$

Proposed by Kostantinos Geronikolas-Greece

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{k=1}^{n} \frac{1}{\sqrt{1+x_k}} = \sum_{k=1}^{n} \frac{1^3}{\sqrt{1+x_k}} \overset{C-B-S}{\underset{(3)}{\geq}} \frac{\left(\sum_1^n 1\right)^3}{n.\sum_{k=1}\sqrt{1+x_k}} \geq$$

$$= \frac{n^3}{n\sqrt{n\left(\sum 1 + \sum x_n\right)}} = \frac{n^2}{\sqrt{n(n+1)}} = \frac{n^{\frac{3}{2}}}{\sqrt{n+1}} = n\sqrt{\frac{n}{n+1}}$$

$$\text{Equality holds when}: \; x_1 = x_2 = x_3 = \dots = \frac{1}{n}$$