

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \prod_{n=0}^{\infty} \left(\frac{1024n^2 + 1024n + 60}{1024n^2 + 1024n + 31} \right) \left(\prod_{n=2}^{\infty} \left(\frac{n^{14} - 2n^8 + n^2 - n^{12} + 2n^6 - 1}{n^{14}} \right) \right)$$

Proposed by Omkumar Rao-India

Solution by proposer

$$\Omega = \prod_{n=0}^{\infty} \frac{\left(n + \frac{1}{16}\right) \left(n + \frac{15}{16}\right)}{\left(n + \frac{1}{32}\right) \left(n + \frac{31}{32}\right)} \left(\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right)^3 \left(1 + \frac{1}{n^2} + \frac{1}{n^4}\right)^2 \right)$$

$$\Omega = \left(\frac{\Gamma\left(\frac{1}{32}\right) \Gamma\left(\frac{31}{32}\right)}{\Gamma\left(\frac{1}{16}\right) \Gamma\left(\frac{15}{16}\right)} \right) \left(\frac{1}{2}\right)^3 \left(\prod_{n=2}^{\infty} \left(1 - \frac{\left(e^{\frac{i\pi}{3}}\right)^2}{n^2}\right) \left(1 - \frac{\left(e^{-\frac{i\pi}{3}}\right)^2}{n^2}\right) \right)^2$$

$$\Omega = \frac{1}{4} \cos\left(\frac{\pi}{32}\right) \left(\left(\frac{\sin\left(\pi e^{\frac{i\pi}{3}}\right)}{\pi e^{\frac{i\pi}{3}}}\right) \cdot \left(\frac{\sin\left(\pi e^{-\frac{i\pi}{3}}\right)}{\pi e^{-\frac{i\pi}{3}}}\right) \right)^2 = \frac{1}{4} \cos\left(\frac{\pi}{32}\right) \frac{\left(\cosh\left(\frac{\sqrt{3}\pi}{2}\right)\right)^4}{9\pi^4}$$

$$\prod_{n=0}^{\infty} \left(\frac{1024n^2 + 1024n + 60}{1024n^2 + 1024n + 31} \right) \left(\prod_{n=2}^{\infty} \left(\frac{n^{14} - 2n^8 + n^2 - n^{12} + 2n^6 - 1}{n^{14}} \right) \right) \\ = \frac{\left(\cosh\left(\frac{\sqrt{3}\pi}{2}\right)\right)^4 \cos\left(\frac{\pi}{32}\right)}{36\pi^4}$$