ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\sum_{n=1}^{\infty}\frac{(2n-1)!!}{2^n(2n)!!}$$

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$$\therefore\ (2n-1)!!=\frac{1\times 2\times 3\ldots\times(2n-1)\times 2n}{2\times 4\times 6\times\ldots\times 2n}=\frac{(2n)!}{2^n\times n!}$$

$$\therefore\ (2n)!!=2^n\times(1\times 2\times 3\times\ldots n)=2^n\times n!$$

$$\therefore\ \frac{(2n)!}{n!\times n!}=C_{2n}^{n}=\binom{2n}{n}\ ,\qquad \sum_{n=0}^{\infty}\binom{2n}{n}\times x^n=\frac{1}{\sqrt{1-4x}},\qquad |x|<\frac{1}{4}$$

$$\Omega=\sum_{n=1}^{\infty}\frac{(2n-1)!!}{2^n(2n)!!}=\sum_{n=1}^{\infty}\frac{\frac{(2n)!}{2^n\times n!}}{2^n\times n!}=\sum_{n=1}^{\infty}\left(\frac{(2n)!}{2^n\times n!}\times\frac{1}{2^n\times n!}\right)=$$

$$=\sum_{n=1}^{\infty}\frac{(2n)!}{2^{3n}\times(n!)^2}=\sum_{n=1}^{\infty}\binom{2n}{n}\times\left(\frac{1}{8}\right)^n=\left(\sum_{n=0}^{\infty}\binom{2n}{n}\times\left(\frac{1}{8}\right)^n\right)-\binom{0}{0}\times\left(\frac{1}{8}\right)^0=$$

$$=\frac{1}{\sqrt{1-4\times\frac{1}{8}}}-1=\sqrt{2}-1$$