

# ROMANIAN MATHEMATICAL MAGAZINE

**Find :**

$$\Omega = \lim_{n \rightarrow \infty} \left( n \int_0^1 \frac{x^n}{1+x^n} dx \right)$$

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*Solution by Omkumar Rao-India*

$$\begin{aligned} n \int_0^1 \frac{x^n}{1+x^n} dx &\stackrel{x^n \rightarrow y}{\rightleftharpoons} n \int_0^1 \frac{y \cdot \frac{1}{n} \cdot y^{\frac{1}{n}-1}}{1+y} dy = \int_0^1 \frac{y^{\frac{1}{n}}}{1+y} dy \\ \Omega &= \lim_{n \rightarrow \infty} \left( \int_0^1 \frac{y^{\frac{1}{n}}}{1+y} dy \right) \stackrel{\text{Dominated Convergence Theorem}}{\rightleftharpoons} \int_0^1 \left( \lim_{n \rightarrow \infty} \frac{y^{\frac{1}{n}}}{1+y} \right) dy = \\ &= \int_0^1 \left( \frac{y^0}{1+y} \right) dy = \int_0^1 \frac{1}{1+y} dy = \ln 2 \end{aligned}$$