ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \lim_{n\to\infty}\left(\lim_{x\to\infty}\left(\underbrace{f\circ f\circ\ldots\circ f}_{\text{"n"}-times}\right)(x)\right),\qquad f(x)=\frac{x}{\sqrt{1+x^2}}$$

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Solution by Daniel Sitaru-Romania

$$(f\circ f)(x)=f(f(x))=f\left(\frac{x}{\sqrt{1+x^2}}\right)=\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+\frac{x^2}{1+x^2}}}=\frac{x}{\sqrt{1+2x^2}}$$

$$(f\circ f\circ f)(x)=f\big((f\circ f)(x)\big)=f\left(\frac{x}{\sqrt{1+2x^2}}\right)=\frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1+\frac{x^2}{1+2x^2}}}=\frac{x}{\sqrt{1+3x^2}}$$

$$P(n):\left(\underbrace{f\circ f\circ\ldots\circ f}_{\text{n}-times}\right)(x)=\frac{x}{\sqrt{1+nx^2}}-suppose\ true$$

$$P(n+1):\left(\underbrace{f\circ f\circ\ldots\circ f}_{\text{n+1}-times}\right)(x)=\frac{x}{\sqrt{1+(n+1)x^2}}-to\ prove$$

$$\left(\underbrace{f\circ f\circ\ldots\circ f}_{\text{n+1}-times}\right)(x)=f\left(\left(\underbrace{f\circ f\circ\ldots\circ f}_{\text{n}-times}\right)(x)\right)=f\left(\frac{x}{\sqrt{1+nx^2}}\right)=$$

$$=\frac{\frac{x}{\sqrt{1+nx^2}}}{\sqrt{1+\frac{x^2}{1+nx^2}}}=\frac{x}{\sqrt{1+(n+1)x^2}}$$

$$P(n)\to P(n+1)$$

$$\Omega=\lim_{n\to\infty}\left(\lim_{x\to\infty}\left(\underbrace{f\circ f\circ\ldots\circ f}_{\text{"n"}-times}\right)(x)\right)=\lim_{n\to\infty}\left(\lim_{x\to\infty}\left(\frac{x}{\sqrt{1+nx^2}}\right)\right)=\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}\right)=0$$