ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\lim_{n\to\infty}\left(\frac{\sqrt{2}\,.\sqrt[4]{4}.\sqrt[8]{8}.\dots.\sqrt[2^n]{2^n}}{n}\right)$$

Proposed by Daniel Sitaru-Romania

Solution by Shirvan Tahirov-Azerbaijan

$$\sqrt{2}\,.\sqrt[4]{4}.\sqrt[8]{8}.\dots.\sqrt[2^n]{2^n}=2^{\sum_{m=1}^{n}\frac{m}{2^m}}$$

$$\therefore\ \sum_{m=0}^{\infty}x^m=\frac{1}{1-x},\ \sum_{m=1}^{\infty}mx^{m-1}=\frac{1}{(1-x)^2}\to\sum_{m=1}^{\infty}mx^m=\frac{x}{(1-x)^2}$$

$$\lim_{n\to\infty}\left(\frac{\sqrt{2}\,.\sqrt[4]{4}.\sqrt[8]{8}.\dots.\sqrt[2^n]{2^n}}{n}\right)=\lim_{n\to\infty}\left(\frac{2^{\sum_{m=1}^{n}\frac{m}{2^m}}}{n}\right)=$$

$$=\lim_{n\to\infty}\left(\frac{2^{\sum_{m=1}^{\infty}\frac{m}{2^m}}}{n}\right)=\lim_{n\to\infty}\left(\frac{2^{\frac{\frac{1}{2}}{\left(1-\frac{1}{2}\right)^2}}}{n}\right)=\lim_{n\to\infty}\left(\frac{2^2}{n}\right)=0$$