ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{k}{k!+(k-1)!+(k-2)!}\right)$$

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Solution by Shirvan Tahirov-Azerbaijan

$$\lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{k}{k(k-1)(k-2)!+(k-1)(k-2)!+(k-2)!}\right)=$$

$$=\lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{k}{(k(k-1)+(k-1)+1)\,.\,(k-2)!}\right)=$$

$$=\lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{k}{(k-2)!\,(k^2-k+k-1+1)}\right)=$$

$$=\lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{k}{(k-2)!\,.\,k^2}\right)=\lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{k-1}{k(k-1)(k-2)!}\right)=$$

$$=\lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{k-1}{k!}\right)=\lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{k}{k!}-\sum_{k=3}^{n}\frac{1}{k!}\right)=$$

$$=\lim_{n\to\infty}\left(\sum_{k=3}^{n}\frac{1}{(k-1)!}-\sum_{k=3}^{n}\frac{1}{k!}\right)=$$

$$=\sum_{k=3}^{\infty}\frac{1}{(k-1)!}-\sum_{k=3}^{\infty}\frac{1}{k!}=(e-1-1)-\left(e-1-1-\frac{1}{2}\right)=\frac{1}{2}$$