ROMANIAN MATHEMATICAL MAGAZINE

$$If\ 0 \le a \le b\ then\ prove\ that:$$

$$\int_a^b \frac{sinx}{x+1}dx \le b - a - \ln\frac{b+1}{a+1}$$

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$Lemma: for\ x > 0\ , sinx < x$

$Proof:$

$$Let\ f(x) = x - sinx\ then\ f'(x) = 1 - cosx \ge 0$$

$$so\ f(x) is\ increasing\ f(x) > f(0) or, x - sinx > 0\ or, x > sinx$$

$$\int_a^b \frac{sinx}{x+1}dx \le \int_a^b \frac{x}{x+1}dx = \int_a^b \left(1 - \frac{1}{x+1}\right)dx =$$

$$= (x)_a^b - (\ln(x+1))_a^b = b - a - \ln\frac{b+1}{a+1}$$

Equality holds for a=b