ROMANIAN MATHEMATICAL MAGAZINE

If $0 < a \leq b$ then:

$$\int_a^b \int_a^b \frac{dxdy}{2+x^2+y^2} \leq \frac{b-a}{2}(arctanb - arctana)$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\frac{1}{2+x^2+y^2} = \frac{1}{(1+x^2)+(1+y^2)} \overset{AM-HM}{\leq} \frac{1}{4}\left(\frac{1}{1+x^2} + \frac{1}{1+y^2}\right)$$

$$\int_a^b \int_a^b \frac{dxdy}{2+x^2+y^2} \leq \frac{1}{4}\int_a^b \int_a^b \left(\frac{1}{1+x^2} + \frac{1}{1+y^2}\right) dxdy =$$

$$= \frac{1}{4}\left(\int_a^b \int_a^b \left(\frac{1}{1+x^2}\right) dxdy + \int_a^b \int_a^b \left(\frac{1}{1+y^2}\right) dxdy\right)$$

$$= \frac{1}{4}\left((y)_a^b(\arctan x)_a^b + (x)_a^b(\arctan y)_a^b\right) = \frac{b-a}{2}(arctanb - arctana)$$

Equality holds for a=b.