

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\infty} \frac{x^3 \tan^{-1} x}{x^8 + x^4 + 1} dx$$

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Solution by Omkumar Rao-India

$$\begin{aligned} \int_0^{\infty} \frac{x^3 \tan^{-1} x}{x^8 + x^4 + 1} dx &\stackrel{x \rightarrow \frac{1}{x}}{\Rightarrow} \int_0^{\infty} \frac{x^3 \left(\frac{\pi}{2} - \tan^{-1} \left(\frac{1}{x} \right) \right)}{x^8 + x^4 + 1} dx \\ 2\Omega &= \frac{\pi}{2} \int_0^{\infty} \frac{x^3}{x^8 + x^4 + 1} dx \stackrel{x^4 \rightarrow x}{\Rightarrow} \frac{\pi}{8} \int_0^{\infty} \frac{1}{x^2 + x + 1} dx \\ \Omega &= \frac{\pi}{16} \int_0^{\infty} \frac{1}{x^2 + x + 1} dx = \frac{\pi}{16} \left(\frac{2\pi}{3\sqrt{3}} \right) \\ \Omega &= \frac{\pi^2 \sqrt{3}}{72} \end{aligned}$$