

ROMANIAN MATHEMATICAL MAGAZINE

Find :

$$\Omega = \int_0^{\infty} \frac{x^2 \sqrt{x} \ln(x)}{x^4 + 1} dx$$

Proposed by Vasile Mircea Popa-Romania

Solution by Omkumar Rao-India

$$I(p) = \int_0^{\infty} \frac{x^p}{x^4 + 1} dx = \frac{\pi}{4 \sin\left(\frac{(p+1)\pi}{4}\right)}, \quad I'(p) = \int_0^{\infty} \frac{x^p \ln x}{x^4 + 1} dx$$

$$I'(p) = -\frac{\pi^2}{16} \cos\left(\frac{(p+1)\pi}{4}\right) \left(\csc\left(\frac{(p+1)\pi}{4}\right)\right)^2$$

$$\Omega = I'\left(\frac{5}{2}\right) = -\frac{\pi^2}{16} \cos\left(\frac{7\pi}{8}\right) \left(\csc\left(\frac{7\pi}{8}\right)\right)^2 = -\frac{\pi^2 \left(-\frac{\sqrt{2+\sqrt{2}}}{2}\right)}{\left(\frac{2-\sqrt{2}}{4}\right)} = \frac{\pi^2}{16} (2+\sqrt{2}) \sqrt{2+\sqrt{2}}$$