ROMANIAN MATHEMATICAL MAGAZINE

Find :

$$\Delta=\int_0^1\int_0^1\frac{x^2+y^2+2}{\sqrt{x^2y^2+x^2+y^2+1}}dxdy$$

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Solution by Shirvan Tahirov-Azerbaijan

$$\Delta=\int_0^1\int_0^1\frac{x^2+y^2+2}{\sqrt{x^2y^2+x^2+y^2+1}}dxdy=\int_0^1\int_0^1\frac{x^2+y^2+1+1}{\sqrt{(x^2+1)(y^2+1)}}dxdy=$$

$$=\int_0^1\int_0^1\frac{(x^2+1)+(y^2+1)}{\sqrt{(x^2+1)(y^2+1)}}dxdy=$$

$$=\int_0^1\int_0^1\frac{(x^2+1)}{\sqrt{(x^2+1)(y^2+1)}}dxdy+\int_0^1\int_0^1\frac{(y^2+1)}{\sqrt{(x^2+1)(y^2+1)}}dxdy=$$

$$=\left(\int_0^1\left(\sqrt{(x^2+1)}\right)dx\times\int_0^1\frac{1}{\sqrt{(y^2+1)}}dy\right)+\left(\int_0^1\left(\sqrt{(y^2+1)}\right)dy\times\int_0^1\frac{1}{\sqrt{(x^2+1)}}dx\right)=$$

$$=\left(\left(\frac{1}{2}\left|x\sqrt{x^2+1}+\ln\left(x+\sqrt{x^2+1}\right)\right|\begin{matrix}1\\0\end{matrix}\right)\times\left|\ln\left(y+\sqrt{y^2+1}\right)\right|\begin{matrix}1\\0\end{matrix}\right)+$$

$$+\left(\left(\frac{1}{2}\left|y\sqrt{y^2+1}+\ln\left(y+\sqrt{y^2+1}\right)\right|\begin{matrix}1\\0\end{matrix}\right)\times\left|\ln\left(x+\sqrt{x^2+1}\right)\right|\begin{matrix}1\\0\end{matrix}\right)=$$

$$=\left(\frac{1}{2}(\sqrt{2}+\ln(1+\sqrt{2}))\times(\ln(1+\sqrt{2})-\ln(1))\right)+\left(\frac{1}{2}(\sqrt{2}+\ln(1+\sqrt{2}))\times(\ln(1+\sqrt{2})-\ln(1))\right)=$$

$$=2\left(\frac{1}{2}(\sqrt{2}+\ln(1+\sqrt{2}))\times\ln(1+\sqrt{2})\right)=(\sqrt{2}+1-1+(\ln(1+\sqrt{2})\times\ln(1+\sqrt{2})))=$$

$$=(\sigma+\ln(\sigma)-1)\times ln^2(\sigma)=\ln(\sigma^\sigma)+ln^2(\sigma)-\ln(\sigma)$$

$\therefore$ ***where σ is the silver ratio***