

ROMANIAN MATHEMATICAL MAGAZINE

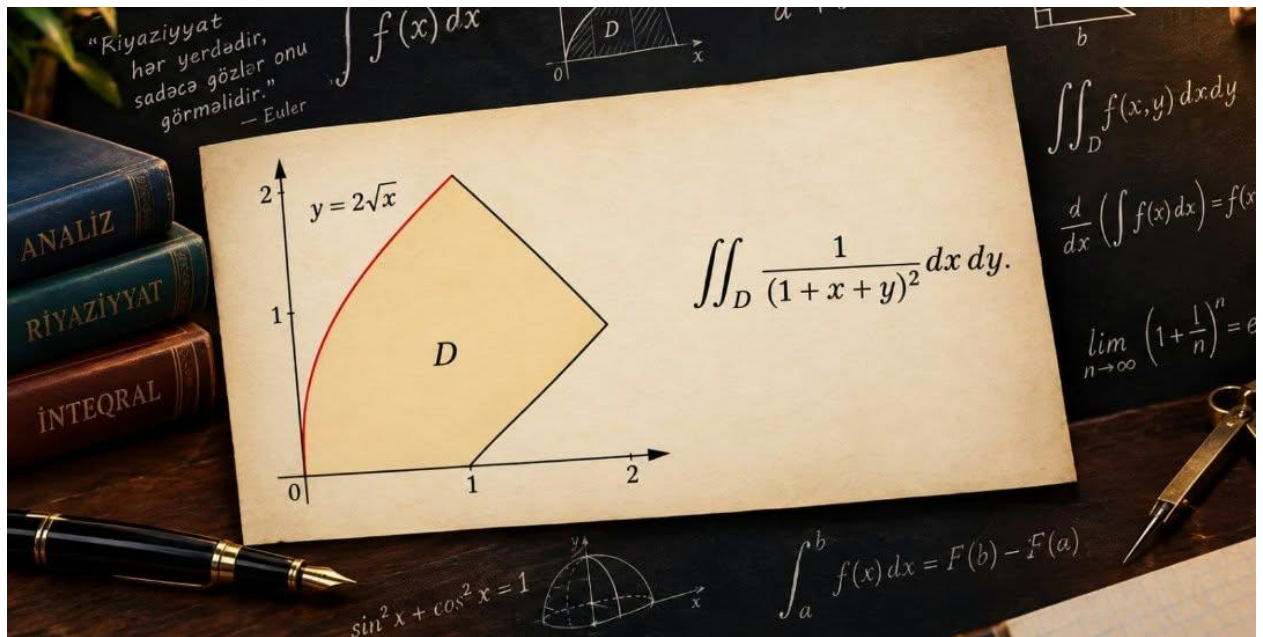
Let be: $D_1 = \{(x, y) | 0 \leq x \leq 1; 0 \leq y \leq 2\sqrt{x}\}$

$D_2 = \{(x, y) | 1 \leq x \leq 2; x - 1 \leq y \leq 3 - x\}$, $D = D_1 \cap D_2$. Find:

$$\Omega = \iint_D \frac{1}{(x + y + 1)^2} dy dx$$

Proposed by Nicola De Rosa-Italy

Solution by Shirvan Tahirov-Azerbaijan



∴ The vertical line extends up ward from $y = 0$ to $y = 2\sqrt{x}$
that is, it is bounded by them :

∴ The vertical line extends up ward from $y = x - 1$ to
 $y = 3 - x$ that is, it is bounded by them :

Let us calculate the given integral using **Fubini's theorem**

$$\therefore \int_m^n \int_k^l f(x, y) dy dx = \int_m^n \left(\int_k^l f(x, y) dy \right) dx$$

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$$\begin{aligned}
 \Omega_1 &= \int_0^1 \int_0^{2\sqrt{x}} \frac{1}{(x+y+1)^2} dy dx = \int_0^1 \left(\int_0^{2\sqrt{x}} \frac{1}{(x+y+1)^2} dy \right) dx = \\
 &= \int_0^1 \left(\left[-\frac{1}{x+y+1} \right]_0^{2\sqrt{x}} \right) dx = \int_0^1 \left(\frac{1}{1+x} - \frac{1}{(1+\sqrt{x})^2} \right) dx = \\
 &= (\ln(2) - 2 \ln(2) - 1) - (\ln(1) - 2 \ln(1)) = 1 - \ln(2) \\
 \Omega_2 &= \int_1^2 \int_{x-1}^{3-x} \frac{1}{(x+y+1)^2} dy dx = \int_1^2 \left(\int_{x-1}^{3-x} \frac{1}{(x+y+1)^2} dy \right) dx = \\
 &= \int_1^2 \left(\left[-\frac{1}{x+y+1} \right]_{x-1}^{3-x} \right) dx = \int_1^2 \left(-\frac{1}{4} + \frac{1}{2x} \right) dx = \\
 &= -\frac{1}{2} + \frac{\ln(2)}{2} - \left(-\frac{1}{4} + \frac{\ln(1)}{2} \right) = -\frac{1}{4} + \frac{\ln(2)}{2} \\
 \Omega &= \Omega_1 + \Omega_2 = (1 - \ln(2)) + \left(-\frac{1}{4} + \frac{\ln(2)}{2} \right) = \frac{3}{4} - \frac{1}{2} \ln(2)
 \end{aligned}$$