ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_{-\pi}^{\pi}\left|2022\sin^2 x-2023\cos^2 x\right|\,dx$$

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$$Let\ \ f(x)=\left|2022\sin^2 x-2023\cos^2 x\right|\ then\ f(-x)=\left|2022\sin^2 x-2023\cos^2 x\right|$$

$$so\ f(x)\ is\ even\ function$$

$$\int_{-\pi}^{\pi}\left|2022\sin^2 x-2023\cos^2 x\right|\,dx=2\int_{0}^{\pi}\left|2022\sin^2 x-2023\cos^2 x\right|\,dx=$$

$$=2\int_{0}^{\frac{\pi}{2}.2}\left|2022\sin^2 x-2023\cos^2 x\right|\,dx=4\int_{0}^{\frac{\pi}{2}}\left|2022\sin^2 x-2023\cos^2 x\right|\,dx=$$

$$=4\int_{0}^{\frac{\pi}{2}}\left|2022\sin^2 x-2023\cos^2 x\right|\,dx=4\int_{0}^{\frac{\pi}{2}}\left|2022-4045\cos^2 x\right|\,dx$$

$$2022\sin^2 x-4045\cos^2 x=0\Rightarrow\ \cos^2 x=\frac{2022}{4045}$$

$$Let\ \cos^2 t=\frac{2022}{4045}\ \ then\ t=\arccos\sqrt{\frac{2022}{4045}},$$

$$sin2t=2sint.cost=\frac{2\sqrt{2022.2023}}{4045}$$

$$when\ x\ increases\ 0\ to\frac{\pi}{2}then\cos^2 x\ decreases\ from\ 1\ to\ 0$$

$$so\ f(x)=<0\ when\ 0\le x<t,\qquad f(x)>0\ \ when\ t\le x<\frac{\pi}{2},f(x)=0\ at\ x=t$$

$$\int_{0}^{\frac{\pi}{2}}\left|2022-4045\cos^2 x\right|\,dx=$$

$$=\int_{0}^{t}\left|2022-4045\cos^2 x\right|\,dx+\int_{t}^{\frac{\pi}{2}}\left|2022-4045\cos^2 x\right|\,dx=$$

$$=\int_{0}^{t}(4045\cos^2 x-2022)dx+\int_{t}^{\frac{\pi}{2}}(2022-4045\cos^2 x)\,dx=$$

$$=\frac{4045}{2}\left(x+\frac{sin2x}{2}\right)_0^t-(2022x)_0^t+(2022x)_t^{\frac{\pi}{2}}-\frac{4045}{2}\left(x+\frac{sin2x}{2}\right)_t^{\frac{\pi}{2}}=$$

$$= \sqrt{2022 \times 2023} + \arccos\sqrt{\frac{2022}{4045}} - \frac{\pi}{4}$$

$(putting\ the\ value\ of\ above\ mentioned\ \ sin2t)$

$$\int_{-\pi}^{\pi} \left|2022 \sin^2 x - 2023 \cos^2 x\right| \ dx = 4\int_{0}^{\frac{\pi}{2}} \left|2022 - 4045 \cos^2 x\right| \ dx =$$

$$= 4\left(\sqrt{2022 \times 2023} + \arccos\sqrt{\frac{2022}{4045}} - \frac{\pi}{4}\right)$$