ROMANIAN MATHEMATICAL MAGAZINE

If $m > 1, n > 0$ then find:

$$\int_0^{\frac{1}{2}} \frac{dx}{m+\sin nx}$$

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Solution by Tapas Das-India

$$Let\ t = nx\ then \frac{dt}{n} = dx\ and\ x = 0 \Rightarrow t = 0\ , x = \frac{1}{2} \Rightarrow t = \frac{n}{2}$$

$$\int_0^{\frac{1}{2}} \frac{dx}{m+\sin nx}. = \frac{1}{n}\int_0^{\frac{n}{2}} \frac{dt}{m+\sin t} = \frac{1}{n}\int_0^{\frac{n}{2}} \frac{1}{m+2\frac{\tan\left(\frac{t}{2}\right)}{1+\tan^2\left(\frac{t}{2}\right)}}dt =$$

$$= \frac{1}{n}\int_0^{\frac{n}{2}} \frac{\sec^2\left(\frac{t}{2}\right)dt}{m+mtan^2\left(\frac{t}{2}\right)+2\tan\left(\frac{t}{2}\right)} = \frac{1}{mn}\int_0^{\frac{n}{2}} \frac{\sec^2\left(\frac{t}{2}\right)dt}{\left(\tan\left(\frac{t}{2}\right)+\frac{1}{m}\right)^2 - \frac{\sqrt{m^2-1}}{m}} =$$

$$= \frac{1}{mn}\int_0^{\frac{n}{2}} \frac{d\left(\tan\left(\frac{t}{2}\right)+\frac{1}{m}\right)}{.\left(\tan\left(\frac{t}{2}\right)+\frac{1}{m}\right)^2 - \frac{\sqrt{m^2-1}}{m}} = \frac{1}{mn}.\frac{m}{\sqrt{m^2-1}}\left(\arctan\left(\frac{mtan\left(\frac{t}{2}\right)+1}{\sqrt{m^2-1}}\right)\right)_0^{\frac{n}{2}} =$$

$$= \frac{1}{n\sqrt{m^2-1}}\left(\arctan\left(\frac{mtan\left(\frac{n}{4}\right)+1}{\sqrt{m^2-1}}\right) - \arctan\left(\frac{1}{\sqrt{m^2-1}}\right)\right)$$