

ROMANIAN MATHEMATICAL MAGAZINE

Find :

$$\Delta = \int_0^1 \int_0^1 \int_0^1 (x + 2y + 3z)e^{x+y+z} dx dy dz$$

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Solution by Odeyemi Gideon-Nigeria

$$\begin{aligned} & \int_0^1 \int_0^1 \int_0^1 (x + 2y + 3z)e^{x+y+z} dx dy dz = \\ &= \int_0^1 \int_0^1 \int_0^1 xe^{x+y+z} dx dy dz + \int_0^1 \int_0^1 \int_0^1 2ye^{x+y+z} dx dy dz + \int_0^1 \int_0^1 \int_0^1 3ze^{x+y+z} dx dy dz = \\ &= (1 + 2 + 3) \int_0^1 \int_0^1 \int_0^1 xe^{x+y+z} dx dy dz \text{ (symmetry) } = \\ &= 6 \int_0^1 \int_0^1 \int_0^1 xe^{x+y+z} dx dy dz = 6 \int_0^1 xe^x dx \cdot \left(\int_0^1 e^y dy \right)^2 = 6(e - 1)^2 \end{aligned}$$