

Find :

$$\Omega = \int_0^{\infty} \frac{1}{x+1+\sqrt[3]{x^2}+\sqrt[3]{x}} dx$$

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Solution by Odeyemi Gideon-Nigeria

$$\sqrt[3]{x} \rightarrow x$$

$$\begin{aligned}\Omega &= 3 \int_0^{\infty} \frac{x^2}{x^3+x+x^2+1} dx = 3 \int_0^{\infty} \frac{x^2}{(x+1)(x^2+1)} dx = \\ &= \frac{3}{2} \left(\int_0^{\infty} \left(\frac{1}{x^2+1} - \frac{2x}{2x^2+2} - \frac{1}{1+x} \right) dx \right) = \\ &= \frac{3}{2} \left(\arctan(x) - \frac{1}{2} \ln(x^2+1) - \ln(1+x) \right) \Big|_0^{\infty} = \\ &= \frac{3}{2} \left(\arctan(x) - \ln \frac{\sqrt{x^2+1}}{x+1} \right) \Big|_0^{\infty} = \frac{3}{2} \left(\frac{\pi}{2} - 0 \right) = \frac{3\pi}{4}\end{aligned}$$