

# ROMANIAN MATHEMATICAL MAGAZINE

**Find:**

$$\Omega = \int_0^{\infty} \frac{dx}{x\sqrt{x} + x + \sqrt{x} + 1}$$

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*Solution by Daniel Sitaru – Romania*

$$\begin{aligned}\Omega &= \int_0^{\infty} \frac{dx}{x\sqrt{x} + x + \sqrt{x} + 1} = \int_0^{\infty} \frac{dx}{(\sqrt{x} + 1)(x + 1)} \stackrel{(x=y^2)}{=} \\&= \int_0^{\infty} \frac{2y dy}{(y + 1)(y^2 + 1)} = \int_0^{\infty} \left( \frac{-1}{y + 1} + \frac{y}{y^2 + 1} + \frac{1}{y^2 + 1} \right) dy = \\&= \left( -\ln(y + 1) + \frac{1}{2} \ln(y^2 + 1) \right) \Big|_0^{\infty} + \arctan y \Big|_0^{\infty} = \\&= \left( \ln \frac{\sqrt{y^2 + 1}}{y + 1} \right) \Big|_0^{\infty} + \arctan \infty - \arctan 0 = \ln 1 - \ln 1 + \frac{\pi}{2} - 0 = \frac{\pi}{2}\end{aligned}$$