ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{secx + sinx}$$

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$$\int \frac{dx}{secx + sinx} = \int \frac{cosx\, dx}{1 + cosx.sinx} = \int \frac{2cosx\, dx}{2 + sin2x} =$$

$$= \int \frac{(cosx + sinx) + (cosx - sinx)}{2 + sin2x} dx =$$

$$= \int \frac{(cosx + sinx)}{2 + sin2x} dx + \int \frac{(cosx - sinx)}{2 + sin2x} dx =$$

$$= \int \frac{(cosx + sinx)}{3 - (sinx - cosx)^2} dx + \int \frac{(cosx - sinx)}{1 + (sinx + cosx)^2} dx =$$

$$= \int \frac{d(sinx - cosx)}{3 - (sinx - cosx)^2} dx + \int \frac{d(sinx + cosx)}{1 + (sinx + cosx)^2} dx =$$

$$= \frac{1}{2\sqrt{3}} \ln\left|\frac{\sqrt{3} + (sinx - cosx)}{\sqrt{3} - (sinx - cosx)}\right| + \arctan(sinx + cosx)$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{secx + sinx} = \left(\frac{1}{2\sqrt{3}} \ln\left|\frac{\sqrt{3} + (sinx - cosx)}{\sqrt{3} - (sinx - cosx)}\right| + \arctan(sinx + cosx)\right)_{\frac{\pi}{6}}^{\frac{\pi}{3}} =$$

$$= \frac{1}{2\sqrt{3}}\left(\ln\left|\frac{3\sqrt{3} - 1}{\sqrt{3} + 1}\right| - \ln\left|\frac{\sqrt{3} + 1}{3\sqrt{3} - 1}\right|\right) = \frac{1}{\sqrt{3}} \ln\left|\frac{3\sqrt{3} - 1}{\sqrt{3} + 1}\right| = \frac{1}{\sqrt{3}} \ln\left|5 - 2\sqrt{3}\right|$$