ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{6}} \frac{sinx}{cosx(1+\sin^2 x)}dx$$

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Solution by Tapas Das-India

$$Let\ cosx = u\ then\ sinxdx = -du$$

$$x = 0 \to u = 1, x = \frac{\pi}{6} \to u = \frac{\sqrt{3}}{2}$$

$$\int_0^{\frac{\pi}{6}} \frac{sinx}{cosx(1+\sin^2 x)}dx = \int_0^{\frac{\pi}{6}} \frac{sinx}{cosx(2-\cos^2 x)}dx = -\int_1^{\frac{\sqrt{3}}{2}} \frac{du}{u(2-u^2)} =$$

$$= \int_{\frac{\sqrt{3}}{2}}^1 \frac{du}{u(2-u^2)} = \frac{1}{2}\int_{\frac{\sqrt{3}}{2}}^1 \frac{u^2+(2-u^2)}{u(2-u^2)}du = \frac{1}{2}\int_{\frac{\sqrt{3}}{2}}^1 \left(\frac{u}{(2-u^2)}+\frac{1}{u}\right)du =$$

$$= -\frac{1}{4}(\ln(2-u^2))_{\frac{\sqrt{3}}{2}}^1 + \frac{1}{2}(\ln(u))_{\frac{\sqrt{3}}{2}}^1 = \frac{1}{4}\ln\frac{5}{4} - \frac{1}{2}\ln\frac{\sqrt{3}}{2} = \frac{1}{4}\ln\frac{5}{4} - \frac{1}{4}\ln\frac{3}{4} = \frac{1}{4}\ln\frac{5}{3}$$