

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos x(1 + \cos x)} dx$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

Let $\cos x = u$ then $\sin x dx = -du$

and $x = 0 \rightarrow u = 1, x = \frac{\pi}{6} \rightarrow u = \frac{\sqrt{3}}{2}$

$$\int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos x(1 + \cos x)} dx = - \int_1^{\frac{\sqrt{3}}{2}} \frac{du}{u(1+u)} = \int_{\frac{\sqrt{3}}{2}}^1 \frac{du}{u(1+u)} = \int_{\frac{\sqrt{3}}{2}}^1 \frac{(1+u) - u}{u(1+u)} du =$$

$$= \int_{\frac{\sqrt{3}}{2}}^1 \frac{du}{u} - \int_{\frac{\sqrt{3}}{2}}^1 \frac{du}{(1+u)} = (\ln u - \ln(1+u)) \Big|_{\frac{\sqrt{3}}{2}}^1 = \ln \frac{2 + \sqrt{3}}{2\sqrt{3}}$$