ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{6}} \frac{sinx}{cosx(1+sinx)}dx$$

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Solution by Tapas Das-India

$$\int \frac{sinx}{cosx(1+sinx)}dx = \int \frac{\text{sinx.cosx}}{\cos^2 x\,(1+sinx)}dx =$$

$$= \int \frac{\text{sinx.cosx}}{(1-\sin^2 x)(1+sinx)}dx \overset{sinx=u}{=} \int \frac{u}{(1-u^2)(1+u)}du =$$

$$= \int \left(\frac{1}{4}\cdot\frac{1}{u+1} - \frac{1}{2}\cdot\frac{1}{(u+1)^2} + \frac{1}{4}\cdot\frac{1}{1-u}\right)du = \frac{1}{4}\ln\left(\frac{1+u}{1-u}\right) + \frac{1}{2}\cdot\frac{1}{1+u} =$$

$$\overset{sinx=u}{=} \frac{1}{4}\ln\left(\frac{1+sinx}{1-sinx}\right) + \frac{1}{2}\cdot\frac{1}{1+sinx}$$

$$\int_0^{\frac{\pi}{6}} \frac{sinx}{cosx(1+sinx)}dx = \left(\frac{1}{4}\ln\left(\frac{1+sinx}{1-sinx}\right) + \frac{1}{2}\cdot\frac{1}{1+sinx}\right)_0^{\frac{\pi}{6}} = \frac{1}{4}\ln 3 - \frac{1}{6}$$